DEPARTMENT OF MATHEMATICS
UNIVERSITY OF ALLAHABAD
ALLAHABAD

UNDERGRADUATE COURSE STRUCTURE

B.A./B.Sc. Part – I

(Effective from Sessioin 2009-10)

There shall be three papers each consisting of 5 units. Each unit will have 14 lectures schedule and hence 70 lectures per paper. Each paper will have 50 marks to its credit. One question with an alternate will be asked from each Unit. Students will have to attempt all questions.

PAPER I : GEOMETRY

Unit – 1

Polar Coordinates :

Polar equation of a parabola, ellipse and hyperbola when focus in taken as pole, Polar equations of the chord joining two points, tangent, normal, polar (chord of contact), pair of tangents, asymptotes, director circle and auxiliary circle of a conic.

Unit – 2

Straight Lines & Planes (Using Vector technique)

Normal form of equation, intercept form of equation and general equation of a plane, plane passing through three points, angle between two planes, two sides of a plane, Length of perpendicular from a point to a plane, Bisectors of angles between two planes, Planes passing through the line of intersection of two planes.

Symmetrical and non-symmetrical form of equations of a line, transformation of non-symmetrical form to symmetrical form, Planes passing through a line, coplanar lines, The shortest distance between two lines, Length of perpendicular from a point to a line, Orthogonal projection of a point and a line on a plane, Lines intersecting two lines, intersection of three planes, volume of a tetrahedron, pair of planes.

Unit — 3

SPHERES

Equation of a sphere, Plane section of a sphere and intersection of two spheres, spheres passing through a circle, tangent plane, plane of contact, polar lines, angle of intersection of two spheres, power of a point, radical plane, line, centre of spheres, coaxial system of spheres, Orthogonal systems of spheres.

Unit— 4

(a) CONES & CYLINDERS

Cones and cylinders with given base, intersection of a cone and a plane passing through the vertex of the cone, tangent lines and planes, reciprocal cones, normal plane passing through a generator of the cone, right circular cones and cylinders.

(b) GENERATING LINES

Ruled surfaces, generating lines of a hyperboloid of one sheet and hyperbolic paraboloid and its properties, generators through a point on the principal elliptic section of hyperboloid of one sheet, $(0, \phi)$ point.on hyperboloid of one sheet and equations of generators at $(0, \phi)$ point.

Unit – 5

CENTRAL CONICOIDS & PARABOLOIDS:

Standard equations of central conicoids and paraboloids, tangent lines and planes, polar planes and polar lines, enveloping cones and cylinders, section with given centre, diametral planes of central conicoids and paraboloids, Locus of point of intersection of three mutually perpendicular tangent planes to central conicoids and paraboloids, Normals drawn from a point to a central conicoid and a paraboloid, Cone passing through the normals drawn from a point to central conicoid and paraboloid, Conjugate diameters of an ellipsoid and its properties.

Books recommended:

1. R. S. Gupta and R. D. Pathak : Conic Sections.
2. Mata Ambar Tiwari and R. S. Sengar : A course in Vector Analysis and its Applications.
3. N. Saran and R. S. Gupta : Analytical Geometry of Three Dimensions.

B.A./B.Sc. Part – I
(Effective from Session 2009-2010)

PAPER – II : ELEMENTARY ANALYSIS

Unit – 1

Statements, Connectives (Conjunction, Disjunction, Negation, Conditional and Biconditional Joins) Statement formulas, Tautologies, Implication and equivalences, Statement Functions, Quantifiers, Sets, Relations, Equivalence and Order relations, Partitions, Functions, Direct and Inverse images of subsets under maps.

Axiomatic introduction of IR as a complete ordered Field, Existence of square roots of positive real numbers.

Unit – 2

Properties of Integers and Sequneces

Natural numbers, First Principle of induction, Well Ordering property of N, Second Principle of Induction, Integers and rational numbers, Archimedean Property, Rational and Irrational Density Theorems, Division and Euclidean Algorithm in Z, Primes, Fundamental Theorem of Arithmetic, Irrationality of surds.

Intervals in IR, Real sequences and their algebra, Limit of a sequence, Bounded, convergent, monotone and Cauchy sequences, Cauchy's general principle of convergene, Algebra of limits (passage to limits under addition, multiplcation, inversions and inequality), Divergence of sequences.

Unit – 3

Limits and Continuity

Real valued Functions of one variable, their graphs and algebra, Neighbourhoods of a point and limit points of subsets of IR, Limit of a function at limit point of its domain, sequential characterization of limit of a function, Algebra of limits, One sided limits, Limit of a function as $x \to \pm\infty$, Infinite limits.

Continuity, Local Boundedness and local maintenance of sign, Boundedness and intermediate value properties of continuous functions over closed intervals, Image of a closed interval under continuous maps.

Unit – 4

Differentiability

Differentiability of a function at a point and its geometrical interpretation, algebra of differentiable functions. Chain rule of differentiation, sign of derivatives and monotonicity of functions, Interior Extremum Theorem, Rolle's Theorem, Lagrange's Mean value Theorem, Cauchy's Mean value Theorem.

Unit – 5

Applications of Derivatives

Higher derivatives, Leibnitz Theorem, Taylor's and Maclaurin's Theorem with Lagrange's and Cauchy's Forms of remainders, Maxima and minima, Local extremum points (necessary and sufficient conditions), Indeterminate Forms, L'Hospital's rule, Convexity and concavity, Points of Inflexion, curvature of curves in explicit form $y = f(x)$, Darboux's Theorem on intermediate value property of derivatives.

Books Recommended

1. Sherbert & Bartle : Introduction to Real Analysis.
2. N.N. Bhattacharya : Elementary Analysis.
3. Thomas & Finney : Calculus & Analytic Geometry.
4. T.M. Apostol : Calculus Vol. 1.
5. Smith & Albrecht : Fundamental Concepts in Analysis.
6. S. C. Malik : Mathematical Analysis.
7. Arora & Bansi Lal : Real Analysis.

B.A./B.Sc. Part – I

(Effective from Session 2009-2010)

PAPER III : DIFFERENTIAL EQUATIONS AND VECTOR CALCULUS

Unit 1

Origin, concept and formation of an ordinary differential equation and its solution, initial-value problems and statement of Existence and Uniqueness Theorems, order and degree of differential equation, First order equations, separable equations, equations reducible to separable form, exact equations.

Integrating factors, linear equations, Geometrical meaning of a different equation and its solution, isoclines and direction field.

Unit – 2

Picard's iteration method, Curves under given geometrical conditions, orthogonal trajectories, mechanical applications (Newton's law of cooling, growth and decay problems, motion in a resisting medium, escape velocity). Equations solvable for p, x or y, Clairaut's equations, singular solutions.

Unit – 3

Homogeneous linear differential equations, linearly dependent and independent solutions, fundamental system of solutions, equations with constant coefficients, Methods of undetermined coefficients, variation of parameters and operators for determining particular integrals, Euler-Cauchy equations.

Unit – 4

(a) Applications to damped and forced oscillations, linear systems of differential equations with constant coefficients.

(b) Derivatives of a vector function of a single scalar variable, scalar and vector fields, gradient, divergence and curl, vector identities.

Unit – 5

Orthogonal curvilinear coordinates, spherical polar and cylindrical polar coordinates double integrals. Line, surface and volume integrals. Green's, Stokes' and Gauss' Theorems (statements only) and their simple applications.

Books Recommended

1. R.S. Sengar : Ordinary Differential Equations.
2. E. Kreyszig : Advanced Engineering Mathematics.
3. B. Rai & D. P. Choudhury : Ordinary Differential Equations : An Introduction.
4. Mata Ambar Tiwari & R.S. Sengar : A Course in Vector Analysis and its Applications.
5. G. F. Simmons : Differential Equations.

B.A./B.Sc. Part – II

(Effective from Session 2010-2011)

There shall be three papers each consisting of 5 units. Each unit will have 14 lectures schedule and hence 70 lectures per paper. Each paper will have 50 marks to its credit. One question with an alternate will be asked from each unit. Students will have to attempt all questions.

Paper – I : LINEAR ALGEBRA

Unit – 1

Definition of a field, Examples R, C, Q and Z_p of fields. Vector Space : definition, examples and basic properties, Subspaces, subspace generated by a subset, Linear dependence and independene of a subset, Basis and Dimension of a vector space, Sum and direct sum of subspaces, Quotient spaces.

Unit – 2

Linear transformations and Isomorphism of vector spaces, Null space, range space, rank and nullity of linear transformations, Fundamental theorem of homomorphism, Rank nullity Theorem, Hom (V, W) as a vector space. Dim Hom (V, W) = Dim V. Dim W. Dual space V^* and transpose of a linear transformation, Annihilator of a subset of a vector space.

Unit – 3

Matrix representation of linear transformation. Algebra of matrices, Effect of change of basis on matrix representation. Equivalent and similar matrices, Rank and nullity of a matrix, System of linear equations, their

consistency and inconsistency Reduction of a matrix to normal form and algorithms to find the inverse (if it exists), Definition of Determinant and statement of properties (without proof), Co-factor, Adjoint of a matrix and its properties.

Unit – 4

Inner product space : Definition and Examples, Cauchy Schwarz inequality, Norm of a vector, Notion of angle, Orthogonality, Orthonormal set and basis, Gram Schmidt orthonormalization, Bessel's Inequality, Orthogonal complement, Adjoint of a linear transformation, inner product and distance preserving transformations (orthogonal and Unitary), Hermitian, skew Hermitian, Symmetric and Skew Symmetric matrices. Invariant Subspaces, Eigenvalues, Eigenvectors, Characteristic polynomials of a Linear transformation, Cayley-Hamilton Theorem.

Unit – 5

Diagonalization of a matrix with distinct Eigenvalues, Orthogonal (Unitary) reduction of real Symmetric (Hermitian) matrices.

Bilinear forms, Equivalence of bilinear forms, matrix representation of Bilinear forms, Quadratic forms, Classification of Symmetric, Skew Symmetric bilinear forms over R and C. Sylvesters law of inertia, Classification of curves and surfaces represented by equations of 2nd degree.

Books Recommended :

1. K. Hoffman and R. Kunze : Linear Algebra.
2. I.N. Herstein : Topics in Algebra, 2nd Edition.
3. Serge Lang : Introduction to Linear Algebra 2nd Edition.
4. Ramji Lal : Algebra Vol. II.
5. M. Artin : Algebra.

B.Sc./B.Sc. Part – II

(Effective from Session 2010-2011)

PAPER – II : ANALYSIS

Unit – 1

Series of Positive Terms

Convergence of Infinite series, addition and removal of terms, nth term test, Cauchy's criterion, Series of positive terms, Comparison Theorem, Comparison Test. Ratio and Root Tests, Comparison of ratios, Raabe's test, Logarithmic ratio test, Cauchy's Condensation test, De Morgan & Bertrand's test, higher logarithmic ratio test.

Unit – 2

General Series

Convergence of arbitrary infinite series, Alternating Series, Absolute and conditional convergence, Abel's and Dirichlet's test, rearrangement of series, Dirichlet's and Riemann's Theorem on rearrangement of absolutely and conditionally convergent series, convolution (Cauchy) product of series, Mertens' Theorem.

Unit – 3

Riemann's Theory of Integration

Step functions and their integrals, upper and lower integrals of a bounded function, Integrable functions, Riemann's condition of integrablity, Properties of integrals, Mean Value Theorem for Integrals, Differentiations of functions defined by integrals, Fundamental Theorem of integral calculus, primitives, change of variables, Second Mean Value Theorem, Integral Test.

Unit – 4

Convergence of Improper Integrals

Integrals over infinite intervals with bounded integrands, Convergence of such integrals, Necessary and sufficient conditions, Case of positive integrands, comparison test, μ-test, absolute convergence, convergence of integrals of product of two functions, Abel's and Dirichlet's test Convergence of integrals with unbounded integrands.

Unit – 5

Metric Spaces

Definition and examples of Metric spaces, Open balls, interior, boundary, closure and accumulation points, Open and closed sets, Interior and closure of sets, Limits of sequences in Metric spaces, Cauchy sequences, Completeness.

Books Recommended

1. S.C. Malik : Mathematical Analysis.
2. Smith & Albrecht : Fundamental Concepts of Analysis.
3. G.F. Simmons : Topology & Modern Analysis.
4. D. Widder : Advanced Calculus.
5. Arora & Bansi Lal : Real Analysis.
6. Shanti Narayan : Real Analysis.
7. T. M. Apostol : Mathematical Analysis.

B.A./B.Sc. – II

PAPER III – MECHANICS

(Effective from Session 2010-2011)

Statics

Unit – 1

Common centenary, virtual work and stability of equilibrium.

Dynamics of a Particle

Unit – 2

Rectilinear Motion : Simple Harmonic Motion including the cases of horizontal and vertical elastic strings, Motion under inverse square law, Motion in resisting media, Motion of varying mass.

Unit – 3

Motion in plane : Kinematics and kinetics of motion, velocity and acceleration in Cartesian, polar and tangential & normal coordinates, determination of path under a given force.

Unit – 4

Constrained Motion : Motion in a vertical circle, Cycloidal motion.

Unit – 5

Motion under Central Forces : Conservation of angular momentum, areal velocity, differential equation to the path of a particle moving under a central attractive force, (p, r) equation to the path, energy equation, Kepler's laws of planetry motion.

Books recommended :

1. S.L. Loney : Statics.
2. P.L. Srivastava : Elementary Dynamics.

B.A./B.Sc. Part – III

(Effective from Session 2011-2012)

There shall be 4 papers each consisting of 5 units. Each unit will have 14 lectures schedule and hence 70 lectures per paper. The first three papers will be of 60 marks credit and the fourth will be of 45 marks credit. One question with an altrnate will be asked from each unit. Students will have to attempt all questions.

PAPER – I : ABSTRACT ALGEBRA

Unit – 1

Semigroups, Groups : definition, elementary properties and Examples including V_4, Q_8, D_8, Z_m, U_m, Group of Symmetries of regular n-gons, Matrix groups and group of Rigid motion of R^n (Example). Homomorphisms, Isomorphisms, (To determine homomorphism between groups given above), Subgroups, Subgroup generated by a subset, Cyclic groups, order of an element. Classification of Cyclic groups, Subgroups of above groups, Group of automorphisms, center of a group, coset decomposition, Lagrange Theorem and applications to number Theory. (Euler's Theorem, Fermat Theorem and Wilson's Theorem).

Unit – 2

Product of groups, Normal Subgroups, Quotient groups, Correspondence theorem, Fundamental Theorem of homomorphisms, 1st and 2nd Isomorphism Theorem and their applications, commutator subgroups, Abelianizer.

Symmetric Groups, Representation of a finite group as a symmetric group and as an orthogonal group, Cycle and Transposition, decomposition of permutations, Alternating

map, Even and Odd permutations, Alternating groups, Normal subgroups of S_n.

Unit – 3

Group action : Definition and Examples including regular and Inner Conjugation action, Orbits and Isotropy groups. Class formula with special reference to Inner conjugation, Conjugacy clases and centralizer, Normalizer and Conjugacy class of subgroups, Prime power order group and its center being non-trivial, Abelian property of groups of order p^2, Classification of groups of order p^2, Classification of groups of order upto 8.

Unit – 4

Rings, Subrings, Homomorphisms, Examples of rings including Z_m, Z[i], Z[w], Matrix rings, Quaternion rings, Ideals, quotient rings, Fundamental Theorem of homomorphism, Correspondence Theorem, Integral domains, characteristic of an integral domain, Division rings and Fields, Field of fractions of a commutative integral domain, Polynomial ring over Commutative ring with identity, and over fields, division algorithm, remainder and factor theorem, Ideals of F[X].

Unit – 5

Linear Congruences and an algorithm to find solutions of linear congruences, group of units of Z_m, Chinese remainder Theorem, quadratic residues, Euler's Criterion, Legendre Symbol, Gauss' Quadratic Reciprocity Theorem. Multiplicativity of Euler's phi function, Divisor function, sum of divisor function and Moebius functions, Moebius Inversion Theorem and its some applications.

Books Recommended :

1. I.N. Herstein : Topics in Algebra, 2nd Edition, Wiley Eastern Ltd. 1988.
2. M. Artin : Algebra, Prentice-Hall of India Pvt. Ltd. New Delhi, 1994.
3. J.B. Fraleigh : Narosa Publishing House, New Delhi 1998.
4. Ramji Lal : Algebra Vol. I, Shail Publication.
5. David M. Burton : Elementary Number Theory, Tata McGraw-Hill Publishing Company Ltd., New Delhi 2007.
6. I. Niven, H. S. Zuckerman, H.L. Montgomery : The Theory of Numbers, John Wiley & Sons, India, 2000.

B.A./B.Sc. – Part III

(Effective from Session 2011-2012)

Paper II : ADVANCED ANALYSIS

Unit – 1

Limits and Continuity

Limits of a function between Metric spaces, Continuity, Characterization of continuity in terms of open sets, closed sets and closures, Continuity of linear transformations and inner product function. Examples from functions between Euclidean spaces, Repeated limits.

Compact Metric spaces, Total Boundedness, Sequential compactness, Lebesgue Covering Lemma, Uniform continuity, Preservation of compactness under continuous maps, Completeness of metric spaces.

Unit – 2

Functions of Several Variables

Partial derivatives, Directional derivatives, differentiability of functions of several variables, Sufficient conditions for continuity and differentiability in terms of partial derivatives, algebra of differentiable functions, Differentiability of composite function, Chain rule of differentiation, Total differentials, Mean value Theorem for real valued functions.

Unit – 3

Applications of Partial differentiation

Homogeneous functions and Euler's Theorem, Equality of Mixed derivatives, Young's & Schwarz' Theorems, Higher differentials, Taylor's Theorem, Maxima and Minima

for real valued functions of several variables. Necessry and sufficient conditions, Saddle points, Lagrange's Multipliers.

Jacobian Matrix and determinants, Implicit Function Theorem (Proof only for $f: \mathbb{R}^2 \to \mathbb{R}$), inverse Function Theorem (Statement only), Functional dependence.

Unit – 4

Uniform Convergence

Pointwise and uniform convergence of sequences and series of functions, Necessary & sufficient conditions. (Cauchy's criterion), Weierstrass', Abel's & Dirichlet's test for uniform convergene. Term by term integration and differentiation of infinite sequences & series of functions.

Power series, Interval of convergence, uniform convergence of power series, Abel's limit theorem, Taylor and Maclaurin series for functions of one variable, Standard functions (sine, cosine, exponential, logarithmic) defined by power series and their properties.

Unit – 5

Complex Analysis

Convergence of Power series over C, Differentiability of functions $f: D \to C$ $(D \subseteq C)$, Analyticity, Cauchy-Riemann equations, Analyticity of functions defined by power series inside the circle of convergence, Complex standard functions exp z, Sinh z, cosh z, cos z, sin z, log z and their representation in terms of real functions.

Books Recommended :

1. S.C. Malik : Mathematical Analysis.
2. G.F. Simmons : Topology & Modern Analysis.

3. T.M. Apostol : Mathematical Analysis.
4. D. Widder : Advanced Calculus.
5. Arora & Bansi Lal : Real Analysis.
6. Shanti Narayan : Mathematical Analysis.
7. Conway : Functions of one complex variable.

B.A./B.Sc. Part – III

(Effective from Session 2011-2012)

Paper III : ADVANCED MECHANICS

Statics (Using Vector Technique)

Forces in three dimensions, Line Coordinates of a line, Central Axis and Wrench, Resultant wrench of two wrenches, Nul line, Nul plane, Nul point with respect to a system of forces, conjugate forces and conjugate lines.

DYNAMICS

Unit – 2

Moment of Interia

Moment and product of inertia of some standard bodies, Principal axes, Momental Ellipsoid of a body.

Unit – 3

(a) D'Alembert's principle

The general equation of motion, Motion of the Centre of inertia and motion relative to the centre of inertia.

(b) Motion about a fixed Axis :

Moment of the effective forces about the axis of rotation, moment of momentum about the axis of rotation, kinetic energy of the body rotating about a fixed axis, equation of motion about axis of rotation, reactions of the axis of rotation.

(c) Velocity and acceleration of a moving particle in cylindrical and spherical polar coordinates.

HYDRODYNAMICS

Unit – 4

(a) Lagrangian and Eulerian approaches, Equation of continuity in different coordinates system, boundary surfaces, velocity potential, stream lines.

(b) Euler's equation of motion, steady motion, Bernoulli's equation, Impulsive motion.

Unit – 5

Motion in two dimensions, stream function, irrotational motion, complex potential, sources and sinks, doublets, image system of a simple source with respect to plane, a circle and a sphere, image system of a doublet with regard to a plane, a circle and a sphere, Circle Theorem.

Books Recommended :

1. Mata Ambar Tiwari & R.S. Sengar : A Course in Vector Analysis and its Applications.
2. S.L. Loney : Dynamics of a Particle and of rigid bodies.
3. A.S. Ramsey : A Treatise on Hydrodynamics.

B.A./B.Sc. III

(Effective from Session 2011-2012)

Paper IV : NUMERICAL METHODS

Unit – 1

Need for numerical methods, floating point representation, rounding off rules.

Solution of arbitrary equations : Fixed point iteration method, Bisection method, Method of false position (Regula-Falsi), Newton-Raphson's method.

Unit – 2

Interpolation : Operators : Δ, ∇, μ, δ, E and D and their relationship, forward, backward and central difference-tables, Gregory-Newton Forward and Backward interpolation formulas, Lagrange's and Newton's Divided difference interpolation formulas, inverse interpolation, Formulas based on central differences : Gauss', Stirling's, Bessel's and Everett's interpolation formulas.

Unit – 3

Cubic spline interpolation, clamped and natural splines.

Numerical Differentiation and Integration : Differentiation using interpolation formulas, Quadrature formulas, Trapezoidal and Simpson's One-third and Three-eight rules.

Unit – 4

Numerical Methods for O.D.E.'s : First order equations, Incremental Methods : Euler's, Taylor series and Improved Euler methods, Runge-Kutta method, Multistep methods,

Predictor corrector pairs : Adam-Bashforth-Moulton's and Milne's formulas, Second order equations : Taylor series and Runge-Kutta-Nystrom Method.

Unit – 5

Numerical Linear Algebra :

Solution of System of equations : Gauss Elimination, LU-factorisation, Cholesky's method, Gauss-Jacobi and Gauss-Seidel iteration formulas, Curve fitting by Least Square method, Estimation of eigenvalues by Gerschgorin circles, Determination of eigenvalues and eigenvectors by iteration (Power method).

Books Recommended :

1. E. Kreyszig : Advanced Engineering Mathematics
2. K. Sankara Rao : Numerical Methods for Scientist and Engineers
3. S.S. Sastry : Introductory Methods of Numerical Analysis
4. Jain, Iyengar & Jain : Numerical Methods for Scientific and Engineering Computation.
5. F.B. Hildebrand : Numerical Analysis